A.4 Problem set A

Problem 1.1: Hadamard gate on a harmonic oscillator

Consider a harmonic oscillator with the Hamiltonian

$$\hat{H} = \hbar\omega \left(\hat{a}^\dagger \hat{a} + \frac{1}{2} \right) \ . \tag{A.13}$$

Consider the following definitions of a logical qubit states:

$$|0\rangle_L = \cos\theta \ |0\rangle \ + \ \sin\theta \ |1\rangle \ , \tag{A.14a}$$
$$|1\rangle_L = \sin\theta \ |0\rangle \ - \ \cos\theta \ |1\rangle \ , \tag{A.14b}$$

where $|\cdot\rangle_L$ denotes the logical qubit states, and $|\cdot\rangle$ denotes the harmonic oscillator energy states.

Find θ and the evolution time t needed to implement the Hadamard gate on the logical qubit.

We give: $\cos\theta + \sin\theta = \sqrt{2} \ \cos(\theta - \pi/4) = \sqrt{2} \ \sin(\theta + \pi/4)$**.**

Problem 1.2: Controlled-NOT on a harmonic oscillator

We aim at implementing a CNOT gate on two logical qubits that are encoded in the energy states of the Harmonic oscillator.

Propose a representation of the logical qubits and give the required evolution time to achieve the target gate.

Problem 1.3: Toffoli gate on a harmonic oscillator

We want to implement the Toffoli gate on three logical qubits. These qubits are encoded in the energy states of the free Hamiltonian of a Harmonic oscillator.

Give the evolution time t and the mapping of the logical qubits to the physical states of the system.