4.1 Problem set 4

Problem 4.1: Rotation gates on a superconducting qubit

In superconducting transmon systems, single-qubit control is achieved by driving the qubit with a microwave signal $s(t) = A(t)\cos(\omega_d t + \phi)$. In the rotating frame of the qubit, this results in the following drive Hamiltonian:

$$\hat{H}_{\text{drive}} = \frac{\hbar\Omega(t)}{2}\left[\cos(\phi)\hat{X} + \sin(\phi)\hat{Y}\right] \tag{4.1}$$

where $\Omega(t)$ is the Rabi frequency, and ϕ is the microwave phase.

An arbitrary rotation on the Bloch sphere is defined by the unitary $R_{\vec{n}}(\theta) = \mathrm{e}^{-i\frac{\theta}{2}\vec{n}\cdot\vec{\sigma}}$.

1. Assume you want to perform a rotation by an angle θ around an axis in the XY-plane defined by an angle ϕ relative to the X-axis. Express the required pulse area $\int \Omega(t)\ \mathrm{d}t$ in terms of θ.
2. How do you modify the microwave pulse to switch from an X-axis rotation to a Y-axis rotation?

Problem 4.2: Hadamard gate

We want to implement the Hadamard gate ($\hat{H}$).

1. Decompose the Hadamard gate into a sequence of Y and X rotations.
2. Describe the microwave pulses (axis and angle) needed for this sequence.

Problem 4.3: CNOT gate

To implement a CNOT, we use the Cross-Resonance (CR) effect, where the Control qubit is driven at the frequency of the Target qubit. The effective interaction Hamiltonian during a CR drive is

$$\hat{H}_{CR} = \frac{\hbar\Omega}{2}(\hat{Z}\otimes\hat{X})\ . \tag{4.2}$$

The evolution operator is $\hat{U}(t) = \mathrm{e}^{-it\hat{H}_{CR}/\hbar}$.

1. Calculate $\hat{U}(t)\,|j,k\rangle$.
2. Does this unitary perform a CNOT ?
3. How to obtain a CNOT gate ?

