2.8 Problem set 2

Problem 2.1: Bloch sphere

Using the general expression of the qubit in spherical coordinates,

$$|\psi\rangle = \cos\frac{\theta}{2}\ |0\rangle\ +\ e^{i\varphi}\sin\frac{\theta}{2}\ |1\rangle\ ,$$

and using the Bloch sphere representation of a qubit, write the expressions of the following states in terms of the computational basis vectors:

1. The north pole.
2. The south pole.
3. A point on the equator.
4. Intersection points between the sphere and the x-axis.
5. Intersection points between the sphere and the y-axis.

Problem 2.2: Bloch sphere: orthogonality

Show that two states represented by two *antipodal*[4] points on the Bloch sphere are orthogonal.

[4]Antipodal points on a sphere are any pair of points on the surface of a sphere that are symmetrical with respect to the center of the sphere.

Problem 2.3: Entangled and Product states (1)

We consider a two-qubit system. The following are some state vectors:

$$|\psi_1\rangle = |01\rangle \qquad |\psi_7\rangle = \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle - |11\rangle)$$

$$|\psi_2\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \qquad |\psi_8\rangle = \frac{1}{\sqrt{3}}|00\rangle + \sqrt{\frac{2}{3}}|11\rangle$$

$$|\psi_3\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle) \qquad |\psi_9\rangle = \frac{1}{2}(|00\rangle + i|01\rangle + i|10\rangle - |11\rangle)$$

$$|\psi_4\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |01\rangle) \qquad |\psi_{10}\rangle = \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$$

$$|\psi_5\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |11\rangle) \qquad |\psi_{11}\rangle = \frac{1}{2}(|00\rangle - |01\rangle + |10\rangle - |11\rangle)$$

$$|\psi_6\rangle = \frac{1}{\sqrt{2}}(|00\rangle - i|01\rangle) \qquad |\psi_{12}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + e^{i\pi/4}|11\rangle\right)$$

Determine which states are *entangled states* and which are *product states*.

Problem 2.4: Entangled and Product states (2)

Repeat **Problem** 2.3 for the following three-qubit states:

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle) \ ,$$
$$|\psi_2\rangle = \frac{1}{\sqrt{3}}(|100\rangle + |010\rangle + |001\rangle) \ ,$$
$$|\psi_3\rangle = \frac{1}{2}(|000\rangle + |010\rangle + |100\rangle + |110\rangle) \ ,$$
$$|\psi_4\rangle = |11\rangle \otimes \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \ ,$$
$$|\psi_5\rangle = \frac{1}{2}(|000\rangle + |001\rangle + |110\rangle + |111\rangle) \ .$$

Problem 2.5: Measuring a bipartite state

1. We prepare a two-qubit system in the state:

$$|\psi\rangle = |00\rangle + |01\rangle - |10\rangle - |11\rangle \ . \tag{2.29}$$

   (a) Normalise the state.

(b) Is this state entangled? Justify your answer.

(c) If we measure the first qubit, what are the possible outcomes and with what probabilities?

(d) What is the post-measurement state of the system?

2. Now we prepare the system in the state:

$$|\phi\rangle = \alpha |00\rangle + \frac{1}{2} |11\rangle \ , \tag{2.30}$$

where α is a complex parameter. Answer the same questions of part 1 above.

3. From your results of the two parts above, what conclusion can you draw regarding the post-measurement state and the nature of the intial state (entangled versus product state)?